8.5 Using Intercept Form

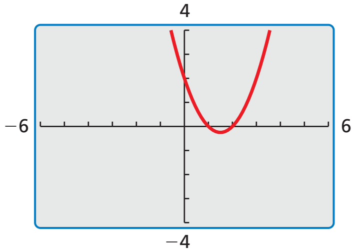
Essential Question

What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?

EXPLORATION 1 Using Zeros to Write Functions

Work with a partner. Each graph represents a function of the form $f(x) = (x - p)(x - q)$ or $f(x) = -(x - p)(x - q)$. Write the function represented by each graph. Explain your reasoning.

a.



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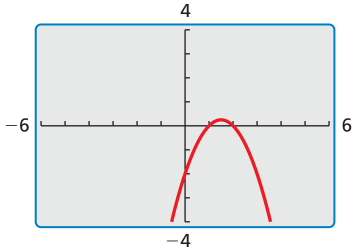

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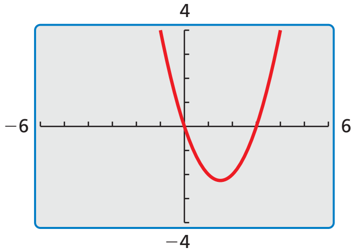

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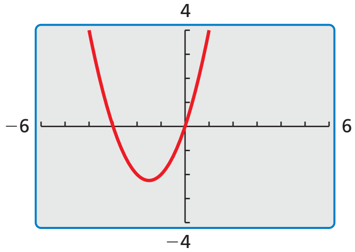

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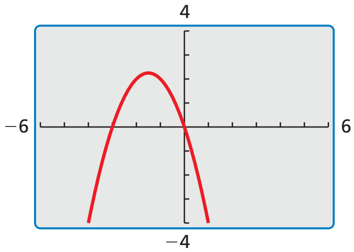

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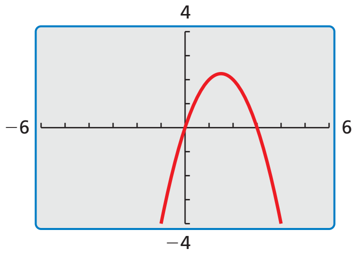
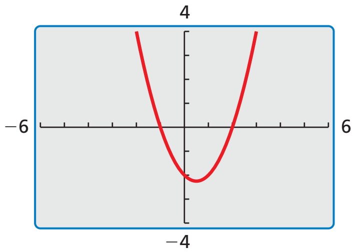
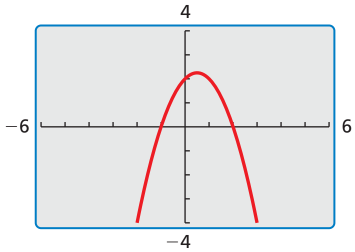


g.



h.



CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

Communicate Your Answer

2. What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?
3. Consider the graph of $f(x) = a(x - p)(x - q)$.
   a. Does changing the sign of a change the x-intercepts? Does changing the sign of a change the y-intercept? Explain your reasoning.
   b. Does changing the value of p change the x-intercepts? Does changing the value of p change the y-intercept? Explain your reasoning.

8.5 Lesson

Core Vocabulary

intercept form, *p. 450*

What You Will Learn

- Graph quadratic functions of the form $f(x) = a(x - p)(x - q)$.
- Use intercept form to find zeros of functions.
- Use characteristics to graph and write quadratic functions.
- Use characteristics to graph and write cubic functions.

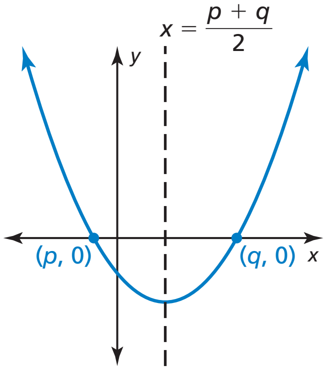
Graphing $f(x) = a(x - p)(x - q)$

You have already graphed quadratic functions written in several different forms, such as $f(x) = ax^2 + bx + c$ (standard form) and $g(x) = a(x - h)^2 + k$ (vertex form). Quadratic functions can also be written in **intercept form**, $f(x) = a(x - p)(x - q)$, where $a \neq 0$. In this form, the polynomial that defines a function is in factored form and the x-intercepts of the graph can be easily determined.

Core Concept

Graphing $f(x) = a(x - p)(x - q)$

- The x-intercepts are p and q.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \dfrac{p + q}{2}$.
- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.



EXAMPLE 1 Graphing $f(x) = a(x - p)(x - q)$

Graph $f(x) = -(x + 1)(x - 5)$. Describe the domain and range.

SOLUTION

Step 1 Identify the x-intercepts. Because the x-intercepts are $p = -1$ and $q = 5$, plot $(-1, 0)$ and $(5, 0)$.

Step 2 Find and graph the axis of symmetry.

$$x = \frac{p + q}{2} = \frac{-1 + 5}{2} = 2$$

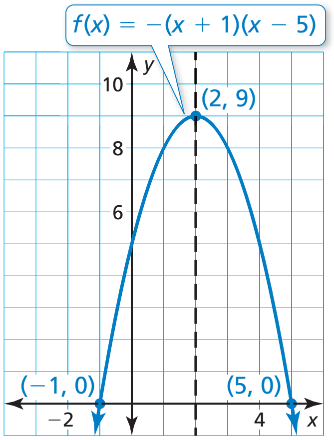
Step 3 Find and plot the vertex.

The x-coordinate of the vertex is 2. To find the y-coordinate of the vertex, substitute 2 for x and simplify.

$$f(2) = -(2 + 1)(2 - 5) = 9$$

So, the vertex is $(2, 9)$.

Step 4 Draw a parabola through the vertex and the points where the x-intercepts occur.



▶ The domain is all real numbers. The range is $y \leq 9$.

EXAMPLE 2 Graphing a Quadratic Function

Graph $f(x) = 2x^2 - 8$. Describe the domain and range.

SOLUTION

Step 1 Rewrite the quadratic function in intercept form.

$$f(x) = 2x^2 - 8 \quad \text{Write the function.}$$
$$= 2(x^2 - 4) \quad \text{Factor out common factor.}$$
$$= 2(x + 2)(x - 2) \quad \text{Difference of two squares pattern}$$

Step 2 Identify the x-intercepts. Because the x-intercepts are $p = -2$ and $q = 2$, plot $(-2, 0)$ and $(2, 0)$.

Step 3 Find and graph the axis of symmetry.

$$x = \frac{p+q}{2} = \frac{-2+2}{2} = 0$$

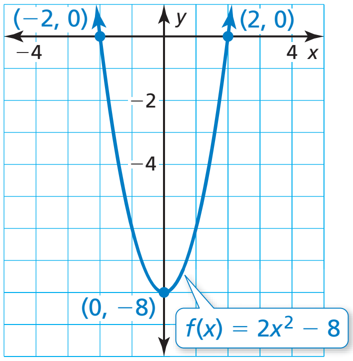
Step 4 Find and plot the vertex.

The x-coordinate of the vertex is 0.
The y-coordinate of the vertex is

$$f(0) = 2(0)^2 - 8 = -8.$$

So, the vertex is $(0, -8)$.

Step 5 Draw a parabola through the vertex and the points where the x-intercepts occur.



▶ The domain is all real numbers. The range is $y \geq -8$.

Monitoring Progress

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Graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function.

1. $f(x) = (x + 2)(x - 3)$ **2.** $g(x) = -2(x - 4)(x + 1)$ **3.** $h(x) = 4x^2 - 36$

REMEMBER

Functions have zeros, and *graphs* have x-intercepts.

Using Intercept Form to Find Zeros of Functions

In Section 8.2, you learned that a zero of a function is an x-value for which $f(x) = 0$. You can use the intercept form of a function to find the zeros of the function.

EXAMPLE 3 Finding Zeros of a Function

Find the zeros of $f(x) = (x - 1)(x + 2)$.

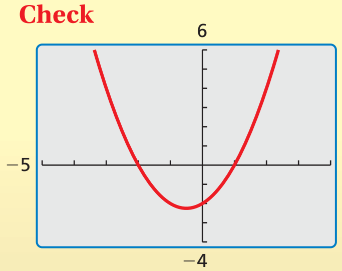
SOLUTION

To find the zeros, determine the x-values for which $f(x)$ is 0.

$$f(x) = (x - 1)(x + 2) \quad \text{Write the function.}$$
$$0 = (x - 1)(x + 2) \quad \text{Substitute 0 for } f(x).$$
$$x - 1 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{Zero-Product Property}$$
$$x = 1 \quad \text{or} \quad x = -2 \quad \text{Solve for } x.$$

▶ So, the zeros of the function are −2 and 1.

Check



Core Concept

Factors and Zeros

For any factor $x - n$ of a polynomial, n is a zero of the function defined by the polynomial.

EXAMPLE 4 **Finding Zeros of Functions**

Find the zeros of each function.

a. $f(x) = -2x^2 - 10x - 12$ **b.** $h(x) = (x - 1)(x^2 - 16)$

SOLUTION

Write each function in intercept form to identify the zeros.

a. $f(x) = -2x^2 - 10x - 12$ Write the function.

$= -2(x^2 + 5x + 6)$ Factor out common factor.

$= -2(x + 3)(x + 2)$ Factor the trinomial.

▶ So, the zeros of the function are -3 and -2.

b. $h(x) = (x - 1)(x^2 - 16)$ Write the function.

$= (x - 1)(x + 4)(x - 4)$ Difference of two squares pattern

▶ So, the zeros of the function are -4, 1, and 4.

LOOKING FOR STRUCTURE

The function in Example 4(b) is called a *cubic function*. You can extend the concept of intercept form to cubic functions. You will graph a cubic function in Example 7.

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Find the zero(s) of the function.

4. $f(x) = (x - 6)(x - 1)$ **5.** $g(x) = 3x^2 - 12x + 12$ **6.** $h(x) = x(x^2 - 1)$

Using Characteristics to Graph and Write Quadratic Functions

EXAMPLE 5 **Graphing a Quadratic Function Using Zeros**

Use zeros to graph $h(x) = x^2 - 2x - 3$.

SOLUTION

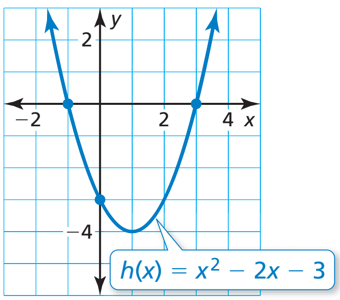
The function is in standard form. You know that the parabola opens up ($a > 0$) and the y-intercept is -3. So, begin by plotting $(0, -3)$.

Notice that the polynomial that defines the function is factorable. So, write the function in intercept form and identify the zeros.

$h(x) = x^2 - 2x - 3$ Write the function.

$= (x + 1)(x - 3)$ Factor the trinomial.

The zeros of the function are -1 and 3. So, plot $(-1, 0)$ and $(3, 0)$. Draw a parabola through the points.



ATTENDING TO PRECISION

To sketch a more precise graph, make a table of values and plot other points on the graph.

EXAMPLE 6 Writing Quadratic Functions

Write a quadratic function in standard form whose graph satisfies the given condition(s).

a. vertex: $(-3, 4)$

b. passes through $(-9, 0)$, $(-2, 0)$, and $(-4, 20)$

SOLUTION

a. Because you know the vertex, use vertex form to write a function.

$f(x) = a(x - h)^2 + k$	Vertex form
$= 1(x + 3)^2 + 4$	Substitute for a, h, and k.
$= x^2 + 6x + 9 + 4$	Find the product $(x + 3)^2$.
$= x^2 + 6x + 13$	Combine like terms.

b. The given points indicate that the x-intercepts are -9 and -2. So, use intercept form to write a function.

$f(x) = a(x - p)(x - q)$	Intercept form
$= a(x + 9)(x + 2)$	Substitute for p and q.

Use the other given point, $(-4, 20)$, to find the value of a.

$20 = a(-4 + 9)(-4 + 2)$	Substitute -4 for x and 20 for $f(x)$.
$20 = a(5)(-2)$	Simplify.
$-2 = a$	Solve for a.

Use the value of a to write the function.

$f(x) = -2(x + 9)(x + 2)$	Substitute -2 for a.
$= -2x^2 - 22x - 36$	Simplify.

STUDY TIP

In part (a), many possible functions satisfy the given condition. The value a can be *any* nonzero number. To allow easier calculations, let $a = 1$. By letting $a = 2$, the resulting function would be $f(x) = 2x^2 + 12x + 22$.

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Use zeros to graph the function.

7. $f(x) = (x - 1)(x - 4)$

8. $g(x) = x^2 + x - 12$

Write a quadratic function in standard form whose graph satisfies the given condition(s).

9. x-intercepts: -1 and 1

10. vertex: $(8, 8)$

11. passes through $(0, 0)$, $(10, 0)$, and $(4, 12)$

12. passes through $(-5, 0)$, $(4, 0)$, and $(3, -16)$

Using Characteristics to Graph and Write Cubic Functions

In Example 4, you extended the concept of intercept form to cubic functions.

$f(x) = a(x - p)(x - q)(x - r),\ a \neq 0$ — Intercept form of a cubic function

The x-intercepts of the graph of f are p, q, and r.

EXAMPLE 7 Graphing a Cubic Function Using Zeros

Use zeros to graph $f(x) = x^3 - 4x$.

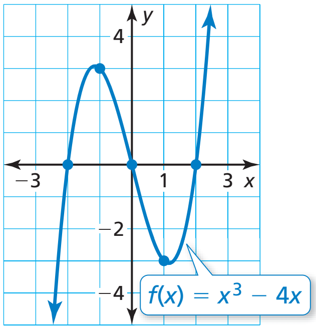
SOLUTION

Notice that the polynomial that defines the function is factorable. So, write the function in intercept form and identify the zeros.

$f(x) = x^3 - 4x$ — Write the function.

$= x(x^2 - 4)$ — Factor out x.

$= x(x + 2)(x - 2)$ — Difference of two squares pattern



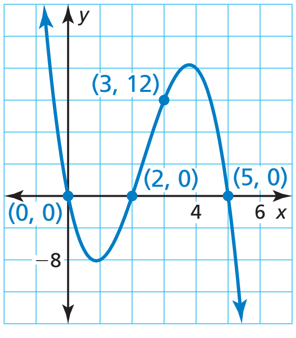
The zeros of the function are −2, 0, and 2. So, plot (−2, 0), (0, 0), and (2, 0).

To help determine the shape of the graph, find points between the zeros.

x	−1	1
$f(x)$	3	−3

Plot (−1, 3) and (1, −3). Draw a smooth curve through the points.

EXAMPLE 8 Writing a Cubic Function



The graph represents a cubic function. Write the function.

SOLUTION

From the graph, you can see that the x-intercepts are 0, 2, and 5. Use intercept form to write a function.

$f(x) = a(x - p)(x - q)(x - r)$ — Intercept form

$= a(x - 0)(x - 2)(x - 5)$ — Substitute for p, q, and r.

$= a(x)(x - 2)(x - 5)$ — Simplify.

Use the other given point, (3, 12), to find the value of a.

$12 = a(3)(3 - 2)(3 - 5)$ — Substitute 3 for x and 12 for $f(x)$.

$-2 = a$ — Solve for a.

Use the value of a to write the function.

$f(x) = -2(x)(x - 2)(x - 5)$ — Substitute −2 for a.

$= -2x^3 + 14x^2 - 20x$ — Simplify.

▶ The function represented by the graph is $f(x) = -2x^3 + 14x^2 - 20x$.

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Use zeros to graph the function.

13. $g(x) = (x - 1)(x - 3)(x + 3)$

14. $h(x) = x^3 - 6x^2 + 5x$

15. The zeros of a cubic function are −3, −1, and 1. The graph of the function passes through the point (0, −3). Write the function.

8.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

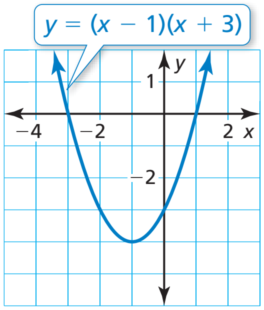
Vocabulary and Core Concept Check

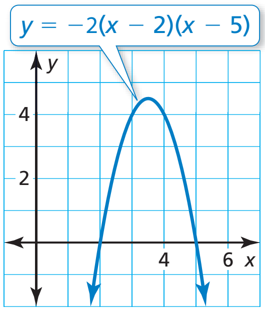
1. **COMPLETE THE SENTENCE** The values p and q are __________ of the graph of the function $f(x) = a(x - p)(x - q)$.

2. **WRITING** Explain how to find the maximum value or minimum value of a quadratic function when the function is given in intercept form.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the x-intercepts and axis of symmetry of the graph of the function.

3. 

4. 

5. $f(x) = -5(x + 7)(x - 5)$
6. $g(x) = \frac{2}{3}x(x + 8)$

In Exercises 7–12, graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function. *(See Example 1.)*

7. $f(x) = (x + 4)(x + 1)$
8. $y = (x - 2)(x + 2)$
9. $y = -(x + 6)(x - 4)$
10. $h(x) = -4(x - 7)(x - 3)$
11. $g(x) = 5(x + 1)(x + 2)$
12. $y = -2(x - 3)(x + 4)$

In Exercises 13–20, graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function. *(See Example 2.)*

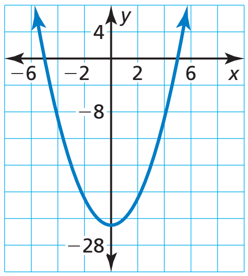
13. $y = x^2 - 9$
14. $f(x) = x^2 - 8x$
15. $h(x) = -5x^2 + 5x$
16. $y = 3x^2 - 48$
17. $q(x) = x^2 + 9x + 14$
18. $p(x) = x^2 + 6x - 27$
19. $y = 4x^2 - 36x + 32$
20. $y = -2x^2 - 4x + 30$

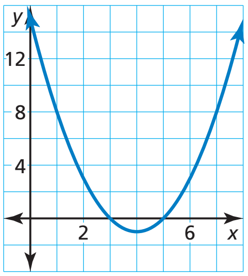
In Exercises 21–30, find the zero(s) of the function. *(See Examples 3 and 4.)*

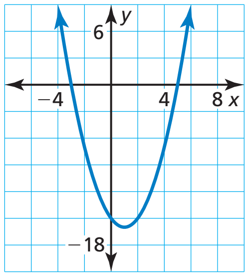
21. $y = -2(x - 2)(x - 10)$
22. $f(x) = \frac{1}{3}(x + 5)(x - 1)$
23. $g(x) = x^2 + 5x - 24$
24. $y = x^2 - 17x + 52$
25. $y = 3x^2 - 15x - 42$
26. $g(x) = -4x^2 - 8x - 4$
27. $f(x) = (x + 5)(x^2 - 4)$
28. $h(x) = (x^2 - 36)(x - 11)$
29. $y = x^3 - 49x$
30. $y = x^3 - x^2 - 9x + 9$

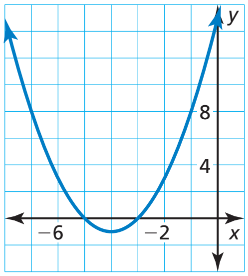
In Exercises 31–36, match the function with its graph.

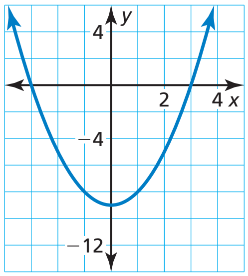
31. $y = (x + 5)(x + 3)$
32. $y = (x + 5)(x - 3)$
33. $y = (x - 5)(x + 3)$
34. $y = (x - 5)(x - 3)$
35. $y = (x + 5)(x - 5)$
36. $y = (x + 3)(x - 3)$

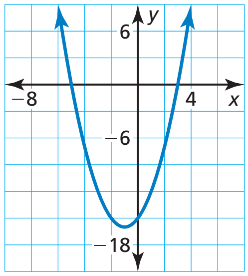
A. 

B. 

C. 

D. 

E. 

F. 

In Exercises 37–42, use zeros to graph the function. *(See Example 5.)*

37. $f(x) = (x + 2)(x - 6)$ **38.** $g(x) = -3(x + 1)(x + 7)$

39. $y = x^2 - 11x + 18$ **40.** $y = x^2 - x - 30$

41. $y = -5x^2 - 10x + 40$ **42.** $h(x) = 8x^2 - 8$

ERROR ANALYSIS **In Exercises 43 and 44, describe and correct the error in finding the zeros of the function.**

43. ✗

$y = 5(x + 3)(x - 2)$

The zeros of the function are 3 and -2.

44. ✗

$y = (x + 4)(x^2 - 9)$

The zeros of the function are -4 and 9.

In Exercises 45–56, write a quadratic function in standard form whose graph satisfies the given condition(s). *(See Example 6.)*

45. vertex: $(7, -3)$ **46.** vertex: $(4, 8)$

47. x-intercepts: 1 and 9 **48.** x-intercepts: -2 and -5

49. passes through $(-4, 0)$, $(3, 0)$, and $(2, -18)$

50. passes through $(-5, 0)$, $(-1, 0)$, and $(-4, 3)$

51. passes through $(7, 0)$

52. passes through $(0, 0)$ and $(6, 0)$

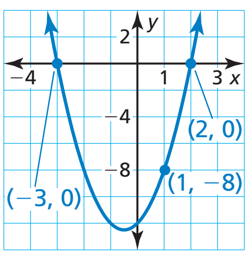
53. axis of symmetry: $x = -5$

54. y increases as x increases when $x < 4$; y decreases as x increases when $x > 4$.

55. range: $y \geq -3$ **56.** range: $y \leq 10$

In Exercises 57–60, write the quadratic function represented by the graph.

57.



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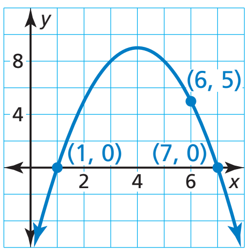

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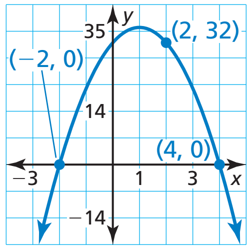

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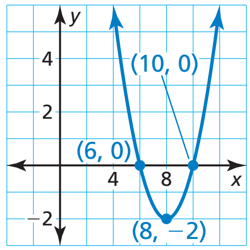


In Exercises 61–68, use zeros to graph the function. *(See Example 7.)*

61. $y = 5x(x + 2)(x - 6)$ **62.** $f(x) = -x(x + 9)(x + 3)$

63. $h(x) = (x - 2)(x + 2)(x + 7)$

64. $y = (x + 1)(x - 5)(x - 4)$

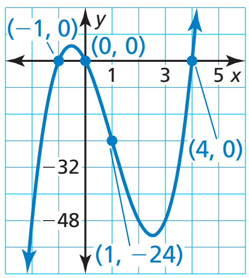
65. $f(x) = 3x^3 - 48x$ **66.** $y = -2x^3 + 20x^2 - 50x$

67. $y = -x^3 - 16x^2 - 28x$

68. $g(x) = 6x^3 + 30x^2 - 36x$

In Exercises 69–72, write the cubic function represented by the graph. *(See Example 8.)*

69.



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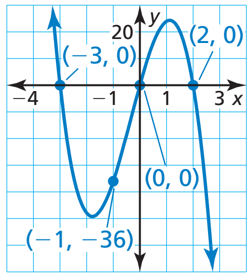
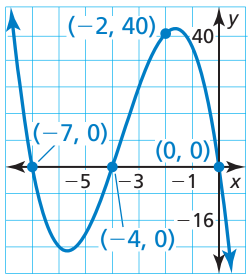
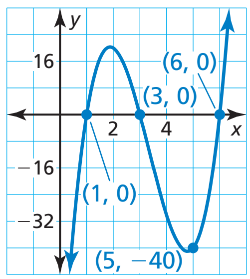


71.



72.



In Exercises 73–76, write a cubic function whose graph satisfies the given condition(s).

73. x-intercepts: -2, 3, and 8

74. x-intercepts: -7, -5, and 0

75. passes through $(1, 0)$ and $(7, 0)$

76. passes through $(0, 6)$

In Exercises 77–80, all the zeros of a function are given. Use the zeros and the other point given to write a quadratic or cubic function represented by the table.

77.

x	y
0	0
2	30
7	0

78.

x	y
-3	0
1	-72
4	0

79.

x	y
-4	0
-3	0
0	-180
3	0

80.

x	y
-8	0
-6	-36
-3	0
0	0

In Exercises 81–84, sketch a parabola that satisfies the given conditions.

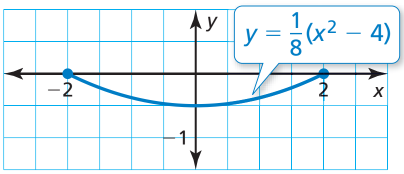
81. x-intercepts: -4 and 2; range: $y \geq -3$

82. axis of symmetry: $x = 6$; passes through (4, 15)

83. range: $y \leq 5$; passes through (0, 2)

84. x-intercept: 6; y-intercept: 1; range: $y \geq -4$

85. **MODELING WITH MATHEMATICS** Satellite dishes are shaped like parabolas to optimally receive signals. The cross section of a satellite dish can be modeled by the function shown, where x and y are measured in feet. The x-axis represents the top of the opening of the dish.

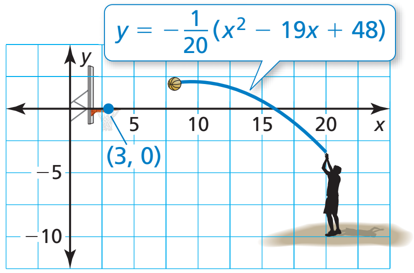


a. How wide is the satellite dish?

b. How deep is the satellite dish?

c. Write a quadratic function in standard form that models the cross section of a satellite dish that is 6 feet wide and 1.5 feet deep.

86. **MODELING WITH MATHEMATICS** A professional basketball player's shot is modeled by the function shown, where x and y are measured in feet.



a. Does the player make the shot? Explain.

b. The basketball player releases another shot from the point (13, 0) and makes the shot. The shot also passes through the point (10, 1.4). Write a quadratic function in standard form that models the path of the shot.

USING STRUCTURE **In Exercises 87–90, match the function with its graph.**

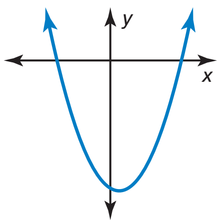
87. $y = -x^2 + 5x$

88. $y = x^2 - x - 12$

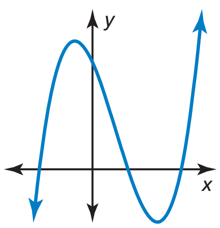
89. $y = x^3 - 2x^2 - 8x$

90. $y = x^3 - 4x^2 - 11x + 30$

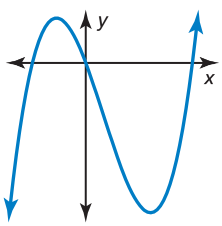
A.



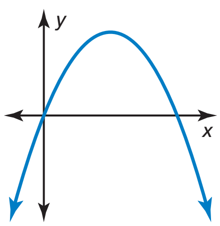
B.



C.



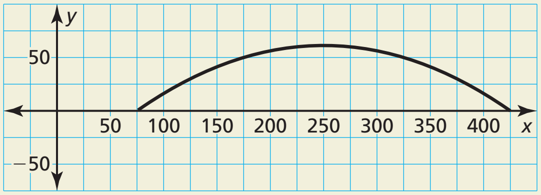
D.



91. **CRITICAL THINKING** Write a quadratic function represented by the table, if possible. If not, explain why.

x	-5	-3	-1	1
y	0	12	4	0

92. HOW DO YOU SEE IT? The graph shows the parabolic arch that supports the roof of a convention center, where x and y are measured in feet.



a. The arch can be represented by a function of the form $f(x) = a(x - p)(x - q)$. Estimate the values of p and q.

b. Estimate the width and height of the arch. Explain how you can use your height estimate to calculate a.

ANALYZING EQUATIONS In Exercises 93 and 94, (a) rewrite the quadratic function in intercept form and (b) graph the function using any method. Explain the method you used.

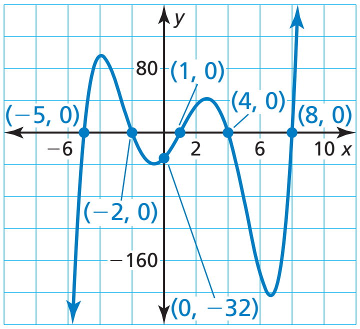
93. $f(x) = -3(x + 1)^2 + 27$

94. $g(x) = 2(x - 1)^2 - 2$

95. WRITING Can a quadratic function with exactly one real zero be written in intercept form? Explain.

96. MAKING AN ARGUMENT Your friend claims that any quadratic function can be written in standard form and in vertex form. Is your friend correct? Explain.

97. PROBLEM SOLVING Write the function represented by the graph in intercept form.



98. THOUGHT PROVOKING Sketch the graph of each function. Explain your procedure.

a. $f(x) = (x^2 - 1)(x^2 - 4)$

b. $g(x) = x(x^2 - 1)(x^2 - 4)$

99. REASONING Let k be a constant. Find the zeros of the function $f(x) = kx^2 - k^2x - 2k^3$ in terms of k.

PROBLEM SOLVING In Exercises 100 and 101, write a system of two quadratic equations whose graphs intersect at the given points. Explain your reasoning.

100. $(-4, 0)$ and $(2, 0)$

101. $(3, 6)$ and $(7, 6)$

Maintaining Mathematical Proficiency

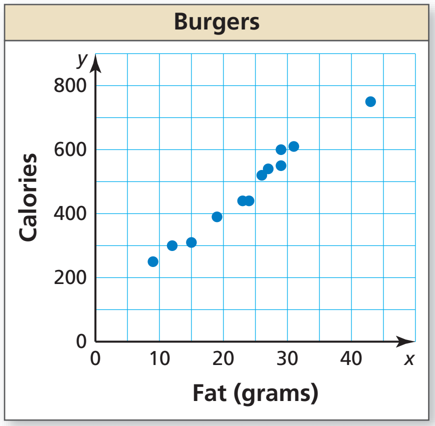
Reviewing what you learned in previous grades and lessons

The scatter plot shows the amounts x (in grams) of fat and the numbers y of calories in 12 burgers at a fast-food restaurant. *(Section 4.4)*

102. How many calories are in the burger that contains 12 grams of fat?

103. How many grams of fat are in the burger that contains 600 calories?

104. What tends to happen to the number of calories as the number of grams of fat increases?



Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning. *(Section 6.6)*

105. 3, 11, 21, 33, 47, . . .

106. −2, −6, −18, −54, . . .

107. 26, 18, 10, 2, −6, . . .

108. 4, 5, 9, 14, 23, . . .